Snap'n'Roll: tuning and listening to the progressive buckling of reticulated ensembles

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Abstract. Progressive snap through-buckling is a catastrophic collapse that is observed in slender and shallow structures such as reticulated roofs or gridshells [1]. It is usually triggered by a local instability and it spreads up to a portion or to the entire structure causing the static failure. To predict the extent of a local snap to the surrounding nodes, the mechanical consequences of local instabilities have to be investigated: apart from static force rearrangement, the impulse of the dynamic rebound can force a domino effect. In this work, each dynamic rebound has been tuned to an input frequency note to compose a simple music sequence with progressive collapses of Von Mises Truss ensembles. Local instabilities have been based on the interaction formulation [2] and fully elastic material and perfect connections have been considered for the whole deformation process. Finally, a comparison between perfect and imperfect systems is presented to translate imperfection sensitivity to musical performance.

Keywords: snap-through · progressive buckling · domes · musical composition · tuned buckling.

1 Introduction

Progressive collapse represents the most severe type of instability mechanism for a reticulated roof [3], [4]. Portions of the structural compartment buckle increasingly, causing, in the worst case, the total failure of the structure. This type of behavior, unpredictable and sudden, is a synonym of structural fragility. If redundancy and ductility are the key ingredients to reduce brittleness in common framed structures [5], structural engineers find it difficult to apply them to domes and gridshells. Redundancy is indeed limited by weight and architectural constraints, while ductility resources would not be engaged since buckling for slender elements usually happens significantly below the elastic limit. It is in fact a local instability, at element or group of element scale, to trigger progressive collapse [6]. It is a radical difference between latticed and continuous shells, where local dents create an energy dissipation area instead of propagating elastically [7]. Emblematic cases of these behaviors are tragically known after the

collapse of the C.W. Post Dome or of the Bucarest Municipality dome [8]. The forensic analyses assessed that the whole structure collapsed (for the case of the Bucarest dome it did dramatically remain hanged and overturned) due to a local and asymmetrical accumulation of the snow. We can imagine a chain reaction-like collapse after the snap-buckling of the members directly loaded by the snow, that progressed through the entire structure. This implies that the design load needs to be assessed locally, and that it can be significantly lower than the total uniformly distributed load that the structure can bear statically (i.e. 1/3 for the case of the C.W. Post [9]).
Treating progressive collapse as a loadcase scenario, the AP (alternate path) method is the standard for structural analysis and design [10]. As the name suggests, it calculates the stability of the reticule once one (or more) buckled members have been removed from the frame. It provides sensitivity indexes as a measure of vulnerability, and it directly depends from a redundancy parameter that takes into account the maximum number or *removable* members before a global collapse. Intuitively, this has to deal with the redistribution and the balancing of the compressive forces inside the frame. Being form-resisting structures, non-isotropic fields translates into non-equilibrium. But static unbalance would not be sufficient to get the entire picture of how the progression advances. Snap-through has a dynamic dimension that directly depends from the motion that structure undergoes reaching the new stable position.
Considering the simplest mechanical model for snap-through in Figure 1a) [11], such motion corresponds to the AB horizontal jump in the equilibrium path.

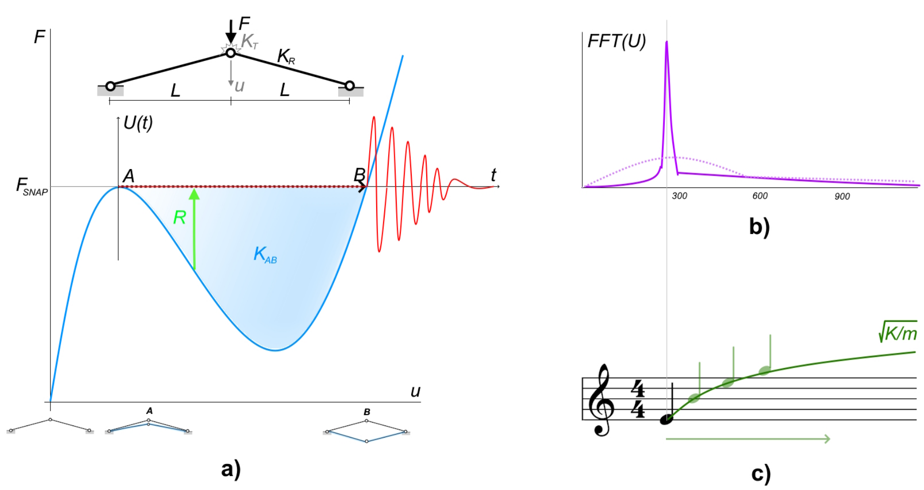

Fig. 1: a) Bellini [11] model for snap-through: equilibrium path (F-u) and post-snap vibrations U(t). b) FFT and frequency identification. c) corresponding FFT(U) note on the staff and linear increase of the stiffness composition

Because position B is a potential energy pit, the arch stops (apart from small oscillations around it) transmitting a dynamic force due to deceleration. The consequential impulsive action occurs also at the lateral restraints, that absorbs both the vertical forces and the vibrations. It is easy to understand that if this is a module of a reticulated roof, those lateral nodes could also snap due to the additional dynamic force on the buckled node, causing a domino effect to the entire structure. Some knockdown factors have been proposed to the AP method to take into account this effect [12], but considering the nonlinearity (and chaotic) nature of the problem, these represent more a blind safety factor than a proper physical counterpart account. Overall, the dynamic effect adds an energy layer to the problem by not treating a quasi-static load application, that has to be dissipated in some way once the system is in a new equilibrium position. Excluding material and connections damage by hypothesis, postbuckling vibrations are the solely dissipating apparatus of the system.
But if vibrations mean frequencies, and frequency means pitch, can we extract a note from the snap-buckling? And what these notes tell once the collapsing mechanism is turn into a musical composition? Examples of a mechanical musical arrangements that have a preset of instructions when externally loaded are the Kircher automata form the 16th century [13]. In Figure 2, one of his designs for a mechanical organ, powered by steam [14]. Carillons works in the same principle, in which notes are produced by the vibrations of plucked bells or accorded foils. Tuned buckling systems have been instead designed for capacitors, sensors and MEMS and have popular and significant applications in current technology [15].

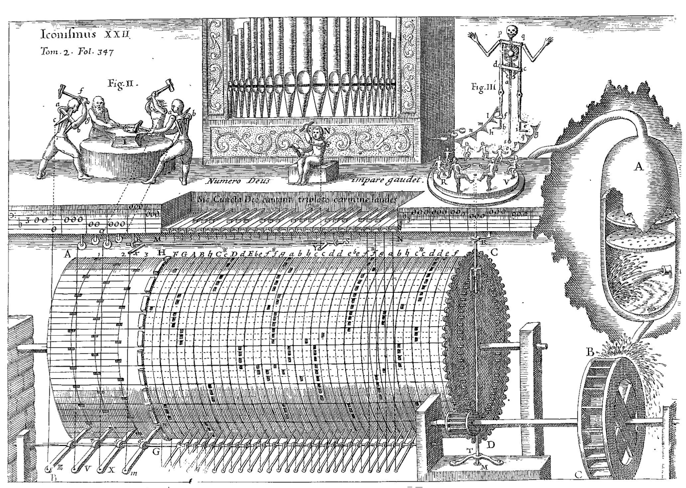

Fig. 2: Athanasius Kircher design for a steam powered mechanical organ [16]

2 Can we play an arch?

The structure in Figure 1a) exhibits a snap-through along the vertical equilibrium path of the central node. Mechanically, the system is governed by the axial stiffness of the rods $K_R = \frac{EA}{L}$, and by the rotational spring stiffness at the crown K_T. If $K_T \neq 0$ the equilibrium path is not symmetrical along the horizontal axis and the snap-through occurs without sign inversion of the vertical reaction F, that can be expressed as follows:

$$F(u) = K_R u(\frac{H}{L}(1+u^2)^{-\frac{1}{2}} - 1) + 2K_T K_R \frac{(\frac{H}{L} - atan(u))}{1+u}. \tag{1}$$

The roots of (1) are the limit points of the path, from which the maximum value of the load F_{SNAP} can be derived. To follow the path after point A, a displacement control is necessary otherwise the system jumps to the next stable position B, at the same level of F_{SNAP} of the hardening branch. This means that for each value of $u \in [A, B]$, $R = F_{SNAP} - F(u)$ is the value of the unbalanced force acting on the system along the unstable curve. Therefore, the blue area in Figure 1a) is the work energy W of the force R, i.e. the elastic energy that will be released by the snap.

$$W(u) = \int R du. \tag{2}$$

Excluding structural damages, this positive gap of energy is the kinematic quote K_{AB} that the system has in point B. Using momentum,

$$W(u) = \int R du = K_{AB} = \frac{1}{2} m(v_B^2 - v_A^2). \tag{3}$$

in where v are the velocities at points A and B and m is the equivalent mass system $m = \frac{P_{SNAP}}{g}$. Because the system is still at point A ($v_A = 0$), final speed at point B can be immediately derived by ,

$$v_B = \sqrt{\frac{2K_{AB}g}{P_{SNAP}}}. \tag{4}$$

Finally, using D'Alembert principal motion equation along vertical axis, oscillations of the system can be derived:

$$\begin{gathered} m\ddot{U} + c\dot{U} + kU = 0, \\ U(t) = \frac{v_B}{\omega}\sin\omega t + u_0 \cos\omega t. \end{gathered} \tag{5}$$

It is worth to note that condensing the structure at the crown node is a simplification allowed solely if this oscillations around position B are relatively small. This means that also that the total stiffness of the system k can be assumed as a constant is the same interval and not as a parametric excitation. Symmetry of the problem leads to

$$k = 2K_R \sin\frac{H}{L} + K_T - K_T = 2\frac{EA}{L}\sin^2\frac{H}{L}. \tag{6}$$

Therefore,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{P_{SNAP}}}. \tag{7}$$

and, as perfect duality with the string vibrations, a higher is the tensile stress (lower the compression), higher is the pitch.

3 Chords and modules

Under this set of assumptions a SDOF system can only play one note[3] because P_{SNAP} is a property and not a control parameter. Thus, to play a sequence of notes additional *snapping nodes* have to be present inside the reticule, i.e. a progressive buckling. Analogously, a simultaneous buckling of multiple nodes would play a musical chord if the resulting frequencies are different. The structure in Figure 3a) aptly describes this behavior. The two-rods archetype above is repeated along the X axis three times at equal distances (green). The central arch is positioned so that the central node (1) has double $Y-$height than the lateral ones in order to realize a double size arch on the perpendicular direction (red). Each node is connected with cylindrical hinges along the perpendicular beam axis. K_T is assumed equal to zero for every node. Built this way, the snap of the central loaded node causes a jump in the vertical forces of the adjacent nodes (2-3) that can consequently lead to their snap. The equilibrium paths of the nodes are shown in Figure 3b). These have been obtained performing a GNIA (Geometrically Nonlinear Analysis with Imperfections) in 200 load increments in LUSAS FEM. Arc-Length control has been activated to follow the postbuckling branches [17]. For clarity, only a small portion after the limit point the equilibrium paths of nodes 2-3 are shown. Point A in the equilibrium path of node 1 represents the first snap-through of the system, causing the overturning of the central portion of the structure as shown by the deform (blue contour) in Figure 3a). The symmetry of the system leads to a simultaneous snap-through of nodes 2-3, as reflected in the equivalent displacement of the equilibrium limit points. Using (7), the produced notes can be obtained as pictures in Figure 3c). The higher pitch of node 2 has been realized by slightly increasing the E of the lateral member. Because the system is designed to not transmit bending moment, it is easy to tune (literally!) E,A or L parameters of the modules by forcing the frequency level on a reference k baseline. The total stiffness k of the system has been obtained by a linear static analysis imposing a unitary displacements to the corresponding notes in the local prebuckling configuration. Interestingly, since

[3] K_T and K_R are actually dependent by internal stress-strain state, thus controlling flexibility to produce large vibration would momentarily rise or low the pitch. Guitarists are use to this for what is called string *bending* technique.

the second layer of snaps occurs with a larger value of P_{SNAP} it produces a lower set of notes. This is intuitively consistent with the mechanic of the arrangement, that has a higher compression at the base level members.

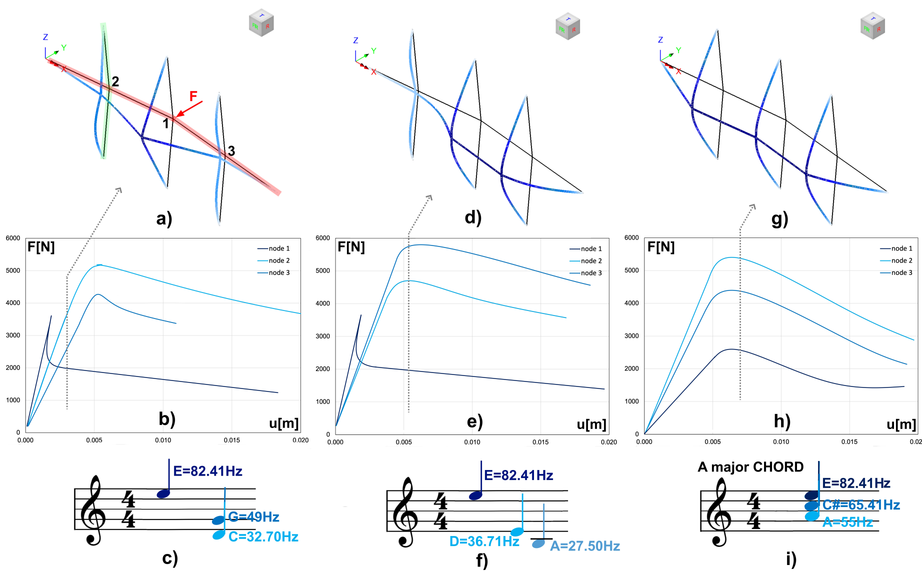

Fig. 3: a) 3 DOF structure, a 4x3 grid realized by 3 Von Mises arches in the Z direction and 1 in the X direction. Y is the vertical axis along acts the central node incremental Force F. In blue contour the deformed shape at the stage indicated by grey dotted line. b) structure equilibrium paths. c) musical composition. d) e) f) and g) h) i) constitutes the same result series.

Using the same logic, the system in Figure 3d) has been tuned to create a *downward movement* musical composition. The deform shown in blue contours describe the vertical displacements field after the second snap-through has happened. A peculiar set of frequencies have instead been chosen for the system in Figure 3g). Once a note is selected, is in fact possible to create a harmonic chord by tuning the notes to be *the fifth* and *the fourth* of *the fundamental*. According to harmonic progression (or Pythagorean) it is possible to build a chord by a frequency ratio of $\frac{3}{2}$ for the fifth and of $\frac{4}{3}$ for the forth in respect to the fundamental note. Table 1 reports the tuned elastic moduli of the members in order to obtain the chord and the simultaneous snap of the three nodes. It is worth to note that for a single-material structural arrangement the same tuning can be performed by varying cross section or length of the members (E has been used in this work for computational simplicity on the nonlinear analyses). It is worth to highlight that musical order correspond to an ordered (and very catastrophic) buckling mechanism for the whole portion. In Figure 3g) the corresponding deformed has been pictured in blue contours.

Table 1: Tuned parameter (Young's modulus) to the corresponding desired notes for the structural arrangements in Figure 3. System c) produces an A major chord (see the fundamental node of 55 Hz).

	ω_1 [Hz]	ω_2 [Hz]	ω_3 [Hz]	$E_1[GPa]$	$\frac{E_2}{E_1}$	$\frac{E_3}{E_1}$
System a)	82.41	49.00	31.70	210E9	1.2	1
System b)	82.41	36.71	27.50	210E9	1.06	0.94
System c)	82.41	65.41	55.00	173E9	1.20	1.23

4 Collapse of a dome

The musical compositions of the progressive buckling of a case study dome are illustrated in Figure 4b. Each musical composition decreasingly corresponds to a shallowness ratio, while the structure is loaded solely at the upper portion. As shown in Figure 4a), the structure has a triangular mesh (diamatic dome) and tubular section of 10 cm and a thickness of 6mm [18] have been employed to realize members. GNIA has been used and a random beam imperfection pattern has been applied by axis deviations from straight members of 1/1000 of the element length. In Figure 4a) the equilibrium path of the dome with the highest shallowness ratio is presented.

This structure evidences in fact the most ordered harmonic composition. The simple metric of absolute distance in terms of frequencies from harmonic progression has been used to detect musical harmony. This result mirrors two structural behavior peculiarities: *(i)* being the flattest geometrical configuration, the meridian is highly susceptible to central loading. Consequently the parallel beam never reach buckling loads and the collapse progresses along the arch-like direction. Furthermore, (ii) in five snaps the dome is completely overturned and hanging from the lateral ring as pointed by point F in the equilibrium path. On the other hand, the higher configurations always present parallel beam collapse [18] and we may assume that they serve as impediments for the snap-through to progress uniformly.

However fascinating, it is necessary to discuss the methodological limitations of the presented results. Firstly, since the dome is not statically determined as the system in the previous Section, it is not possible to easily separate stiffness contributions and use Equation 7. It has been instead numerically computed the area of the equilibrium path as previously presented in Figure 1. Each produced frequency has been then derived by combining 4 and 7, assuming elastic prebuckling node stiffness as value for k. This could be not significantly distant from the actual value but further analyses should be performed to prove it.

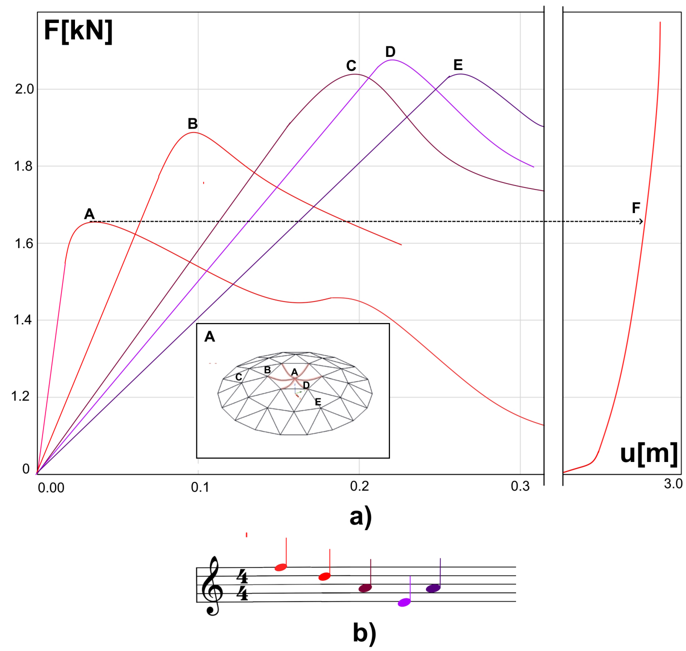

Fig. 4: a) Equilibrium paths of the diamatic dome in the small box. The dotted line indicates the snap-through of the crown node - at point F the dome is completely overturned [18]. b) Corresponding musical composition.

Moreover, considering a twelve tone scale, a difference in frequency higher than 8.3% produces a different note.

5 Conclusion

A musical flavor has been introduced to the classical problem of snap-through instability. In addition to the feasibility of a mechanical instrument, the results can serve as interpretation for snap patterns of progressive instability. Musical order is in fact highly coded by metrics and rules, therefore it may serve as reference model for structural performance or collapse identification. Concerning this, the interpretation of a progressive collapse case study has been presented. All limitations aside, the methodology presented grasped the most severe collapse mechanism using a simple musical disorder metric. Finally, a set of rules to tune a statically determined system to play notes and chords has been presented in Section 2.

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